

Probability is of the study of calculating of the chances an event may occur (does not imply an event will occur)

For a die, it is common knowledge that it has six sides, and you are just as likely to roll each side of equal chance. So

$$P(1) = \frac{1}{6}, P(2) = \frac{1}{6}, \dots, P(6) = \frac{1}{6}$$

Does that mean that when you roll a die 6 times you will roll all six numbers?

Experiment- Roll a die six times and record any numbers that were missing from the list you got.

Roll a die six times and record the results. Identify any number that did not show up in the set.

What is the actual probability of rolling a die and getting all six numbers in six rolls.

First Roll- Can be any of the 6 numbers

Second Roll- Can be any of the 5 numbers that did not occur in the 1st roll

Third Roll- Can be any of the 4 numbers that had not occurred previously

...

Sixth Roll- can only be the roll of the left over number

What is the actually probability of this event.

Since the die are independent, there are 6^6 possible outcomes

The first roll can be 6 possible outcomes, second roll can be 5 possible outcomes, third roll 4, fourth 3, fifth 2, and sixth 1. All six numbers will appear in six rolls a total of 6! Possible ways.

Calculate, what is

$$\frac{6!}{6}$$

What was the percentage? So are you likely to roll all six numbers in six rolls?

Is a family of two always have a boy or a girl

Every time you flip a coin and get heads does that mean the second flip will be tails.

Probability doesn't say that you after 6 rolls you will get all six numbers, it says that for this one event, there is a $\frac{1}{6}$ chance that the number may appear in one procedure.

Recap

Vocab-

An **event** is any collection of results or outcomes of a procedure

A **simple event** is an outcome or event that cannot be broken down further

A **sample space** for a procedure consists of all possible simple events. That is the sample space consists of all possible outcomes that cannot be broken down further.

Relative Frequency Approximation of Probability- Conduct or observe a procedure and count the number of times event A occurs

$$P(A) = \frac{\text{number of times event A occurred}}{\text{number of times procedure was repeated}}$$

Classical Approach to Probability (Requires equally likely outcomes)- Assumes that a given procedure has n different possible outcomes with equal chance of occurring

$$P(A) = \frac{\text{number of ways A occur}}{\text{number of simple events}} = \frac{s}{n}$$

Subjective probability- P(A), the probability event A, is estimated by using knowledge of the relevant circumstances

Law of Large numbers- As a procedure is repeated many times, the relative frequency probability of an event tends to approach the actual probability

Unlikely- An event is unlikely if it's probability is very small, such as .05 or less.

Unusually high/low number- When the outcomes is far from what was expected (ch 5 will go more in-depth)

Compound Event- Any event combining two or more simple events. (or events)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

A occurs
B occurs
A and B occurs

↑
REMOVES
any values
counted 2x

Disjoint Events (mutually exclusive)- Event A and B can not occur at the same time

Complement- If A is an event, then not A is $\bar{A}$

Properties of Complements-

$$P(A) + P(\bar{A}) = 1$$

$$P(A) = 1 - P(\bar{A})$$

$$P(\bar{A}) = 1 - P(A)$$

"And" Events- Usually involve multiplication

$$P(A \text{ and } B) = P(A)P(B|A)$$

A and B
occurs

↑ event B occurs
given A has
happened

Independent Events- When two event A and B are independent, then the occurrence of event A doesn't effect B and the occurrence of event B does not effect A. Below is the equation for independent events

$$P(A \text{ and } B) = P(A)P(B)$$

For large population when finding the probability of a small portion of the population you may treat the events as independent.

At least one event- If an event occurs at least once in N trials. To calculate the $P(\text{"at least once"})$, you would need to find the probability of $P(1 \text{ or } 2 \text{ or } \dots N)$, but it would take too long.

Instead use the complement rule. The complement is "not at least one time" or simply it never happens in N trials. It is easier to calculate and you get the following.

$$P(A) = 1 - P(\bar{A})$$



$$P(\text{"event happens at least once"}) = 1 - P(\overline{\text{"event happens at least once"}})$$



$$P(\text{"event happens at least once"}) = 1 - P(\text{"event never happens in } N \text{ trials"})$$

Conditional Events- Event B taken into consideration that event A has already happened

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

symbol
means
given

Fundamental Counting Rule- Number of ways two events can occur, given that the first event can happen m ways and the second event can happen n way, is calculated by mn .

Factorial Rule- $n!$ = Number of different permutation (order matters!) that n different items can be selected n times (all items are selected).

Permutation Rule- ${}_nP_r = \frac{n!}{(n-r)!}$, when you have n different items and you only select r . (Order matters)

Combination Rule- ${}_nC_r = \frac{n!}{(n-r)!r!}$, Number of different combinations (order doesn't matter)